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E_6 Unification, Large Neutrino Mixings, and SUSY Flavor Problem

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This study indicates that in E_6 grand unified theories (GUTs), once a hierarchical structure of up-quark-type Yukawa couplings is given as a basic structure of flavor, larger lepton mixings than the quark mixings and milder down-quark-type (and charged-lepton-type) mass hierarchies than up-quark-type mass hierarchy can generically be obtained under a few natural assumptions. The basic flavor structure is compatible with non-Abelian horizontal symmetry, which can solve the SUSY flavor problem. It is shown that in solving the SUSY flavor problem, the E_6 structure, which realizes bi-large neutrino mixings, also solves a problem that results from the large neutrino mixing angles.

INTRODUCTION

Several recent neutrino experiments have reported that the neutrino mixings are quite large [1, 2], which are much different from those in the quark sector. Based on the fact that these results are different from previous ideas that the mixings in the lepton sector must be small as those in the quark sector, various grand unified scenarios have been examined in the literature [3, 4, 5, 6, 7, 9, 10].

One of the key observations in understanding the difference between quark mixings and lepton mixings is that quark mixings are determined by the diagonalizing matrices of the representation field $\mathbf{10}(\ni Q = (U_L, D_L), U_R^c, E_R^c)$ of $SU(5)$ that includes doublet quark Q, whereas lepton mixings are determined by those of $\bar{\mathbf{5}}(\ni D_R^c, L = (N_L, E_L))$ that includes doublet lepton L. Therefore, in the context of $SU(5)$ grand unified theories (GUTs), it is not difficult to obtain different mixing matrices of quarks and leptons [3].

$SO(10)$ unification is interesting because all one generation quarks and leptons, including the right-handed neutrino N_R^c, can be unified into a single multiplet $\mathbf{16}$, which is divided as $\mathbf{16} \to \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ under $SO(10) \supset SU(5)$. However, because $SO(10)$ includes $SU(2)_R$, which rotates right-handed quarks (U_R^c, D_R^c) and right-handed leptons (N_R^c, E_R^c), $SO(10)$ symmetric Yukawa interactions, for example, $Y_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H$ $(i = 1, 2, 3)$ with a Higgs field $\mathbf{10}_H$, lead to the same Yukawa matrices for up and down quark sectors. However, these are neither realistic nor consistent with non-vanishing values of the quark mixings in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Therefore, to obtain realistic mass matrices, we have to pick up a non-vanishing vacuum expectation value (VEV) of a Higgs field C (representation $\mathbf{16}$ is the simplest one), which breaks the $SU(2)_R$, to obtain the realistic Yukawa couplings. Moreover, in the high-scale supersymmetry (SUSY) breaking scenario, because of $SO(10)$ relations $Y_u = Y_{\nu_D}$, where Y_{ν_D} is the neutrino Yukawa matrix for Dirac neutrino masses and Y_u is the up-quark-type Yukawa matrix one of whose components is large to obtain large top-quark mass, large mixings in lepton sector generically result in too large $\mu \to e\gamma$ process through loop corrections [11]. One of the most effective ways to avoid these problems is to introduce several matter fields $\mathbf{10}_a$ $(a = 1, \cdots, n)$ of $SO(10)$, which are divided as $\mathbf{10} \to \mathbf{5} + \bar{\mathbf{5}}$ under $SO(10) \supset SU(5)$, in addition to three $\mathbf{16}_i$. Then the mass matrix of n $\mathbf{5}$s and $n+3$ $\bar{\mathbf{5}}$s can pick up the VEV of C through the interactions $\mathbf{16}_i\mathbf{10}_a C$ if we take C as $\mathbf{16}$ of $SO(10)$ whose VEV breaks $SO(10)$ into $SU(5)$. This structure not only avoids the relation $Y_{\nu_D} = Y_u$ but also realizes the difference between the mixing matrices of the quark and lepton sectors. Actually, large neutrino mixings, small CKM mixings, and small Dirac neutrino masses can be obtained in some models [5] using this mechanism.

Then, E_6 unification becomes more interesting, because the fundamental representation $\mathbf{27}$ includes $\mathbf{10}$ as well as $\mathbf{16}$ of $SO(10)$ as

$$\mathbf{27} \to \underbrace{[\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}]}_{\mathbf{16}} + \underbrace{[\bar{\mathbf{5}} + \mathbf{5}]}_{\mathbf{10}} + \underbrace{[\mathbf{1}]}_{\mathbf{1}} \tag{1}$$

under $E_6 \supset SO(10) \supset SU(5)$. Here, the representations of $SO(10)$ and $SU(5)$ are explicitly denoted. In the literature, it has been shown that it is possible to obtain not only large neutrino mixing for the atmospheric neutrino anomaly [6] but also that for the solar neutrino problem [7] in E_6 unification.

It is noteworthy that in Refs. [5], [7], and [12] generic interactions (even for higher dimensional interactions) are introduced with $O(1)$ coefficients not only for the Yukawa interactions [7] but also for the Higgs potentials [12]. Therefore, in the scenario, not only the Yukawa interactions but also the scales of VEVs are determined only by the symmetry of the models.

However, most of the above-mentioned studies concentrated on investigating whether it is possible to obtain large mixings in lepton sector and small mixings in quark sector and did not investigate the reasons for which the lepton sector has larger mixings than the quark sector. One of the purposes of this paper is to clarify this phenomenon. We attribute this phenomenon to "E_6 unification."

Moreover, we emphasize that in the E_6 GUTs, introducing non-Abelian horizontal symmetry $SU(2)_H$ or $SU(3)_H$ naturally solves the SUSY flavor problem. That is, the realistic hierarchical structure of quark and lepton mass matrices can be obtained, and almost universal scalar fermion masses, which are important in suppressing flavor changing neutral current (FCNC) processes, are obtained because at least the first two generation fields are unified into a single multiplet. If $SU(3)_H$ is adopted as the horizontal symmetry, all three generation quarks and leptons can be unified into a single multiplet $(\mathbf{27},\mathbf{3})$. Of course, the idea that non-Abelian horizontal symmetry is introduced to solve the SUSY flavor problem is not new [9, 10, 13];however, this paper emphasizes the fact that the structure peculiar to E_6 GUTs, which realizes bi-large neutrino mixings, also plays an important role in suppressing the FCNC processes sufficiently.

The essential arguments of this paper are similar to those in our previous papers [7, 10] in which anomalous $U(1)$ symmetry [14] is adopted. However, in this paper, we show that most of the arguments can be generally applied to E_6 GUT even without anomalous $U(1)_A$ symmetry and independent of the origin of the Yukawa hierarchy and the mechanism for determination of VEVs. Further, we examine the conditions to realize the above situations.

BASIC ASSUMPTION

E_6 unification [15] is considered to be attractive because the gauge anomaly is automatically free, and E_6 is the maximal exceptional group that has complex representation. Further, all the basic fermions of three generations are unified into three fundamental representation fields $\Psi_i(\mathbf{27})$, which are divided as

$$\Psi_i(\mathbf{27}) \to \mathbf{16}_{\Psi_i} + \mathbf{10}_{\Psi_i} + \mathbf{1}_{\Psi_i} \tag{2}$$

under $E_6 \supset SO(10)$.

It has been argued that the additional component fields $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$ play an important role in realizing rich structure in Yukawa couplings [16] and in realizing large neutrino mixings [6, 7, 8], because three of six $\bar{\mathbf{5}}$ fields become superheavy and light fields are linear combinations of these six $\bar{\mathbf{5}}$ fields. In order to estimate the 3×6 mass matrix of three $\mathbf{5}$ fields and six $\bar{\mathbf{5}}$ fields, we have to fix a part of the Higgs sector that breaks E_6 into the standard model (SM) gauge group $G_{\rm SM} = SU(3)_C\times SU(2)_L\times U(1)_Y$. Supposing that Higgs fields, $\Phi(\mathbf{27})$ and $\bar\Phi(\overline{\mathbf{27}})$, break E_6 into $SO(10)$ ($|\langle \mathbf{1}_\Phi\rangle| = |\langle \mathbf{1}_{\bar\Phi}\rangle|$ to satisfy the D-flatness conditions), $C(\mathbf{27})$ and $\bar C(\overline{\mathbf{27}})$ break $SO(10)$ into $SU(5)$ ($|\langle \mathbf{16}_C\rangle| = |\langle \overline{\mathbf{16}}_{\bar C}\rangle|$ to satisfy the D-flatness conditions), and an adjoint Higgs $A(\mathbf{78})$ breaks $SU(5)$ into the standard model (SM) gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. (Here, we do not fix the direction of the VEV of $\langle A\rangle$ and the scale of $\langle A\rangle$, which may be larger than the VEVs of Φ or C, because it is independent of the following arguments.) Then, through the interactions

$$W_Y = Y_{ij}^\Phi \Psi_i\Psi_j\Phi + Y_{ij}^C\Psi_i\Psi_j C, \tag{3}$$

the mass matrix of $\mathbf{5}$ and $\bar{\mathbf{5}}$ is determined by developing the VEVs of Φ and C.

In the following calculations, for simplicity, we assume that the main modes of the SM doublet Higgs H_u and H_d come from $\mathbf{10}_\Phi$ and $\mathbf{10}_C$. Therefore, the Yukawa couplings for the up-quark sector are essentially determined by the Yukawa couplings Y^Φ and Y^C. However, including the Higgs mixings with $\mathbf{16}_\Phi$ or $\mathbf{16}_C$ does not change the following conclusion drastically unless these $\mathbf{16}$ components dominate the $\mathbf{10}$ components.

One of the most important basic assumptions is as follows. The Yukawa matrices Y^Φ and Y^C have a hierarchical structure that can realize the hierarchy in the up-quark sector. In the literature [9, 17, 18, 19, 20], several mechanisms have been proposed to understand the Yukawa hierarchies. Here, we do not fix the mechanism that realizes such a hierarchical structure. However, we simply assume that both the hierarchies of each of the two Yukawa matrices, Y^Φ and Y^C, have the same origin, that is,

$$(Y^C)_{ij} \sim (Y^\Phi)_{ij} \equiv Y_{ij}. \tag{4}$$

More precisely, we assume that the order of each component of Y^Φ is the same as that of the corresponding component of Y^C, but generally $(Y^\Phi)_{ij} \neq (Y^C)_{ij}$.

QUARK AND LEPTON MASS MATRICES

For simplicity, in the following argument, we adopt

$$Y_{ij} \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \text{ or } \left[\begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\right], \tag{5}$$

where we take $\lambda \sim \sin\theta_C \sim 0.22$. The former is called type A and the latter is type B in this paper. Note that the following arguments can be applied to various other types of matrices that can obtain a realistic up-type quark mass matrix. Once we fix this basic structure of the Yukawa couplings Y, the mass matrix of $(\mathbf{5},\bar{\mathbf{5}})$ fields is given by

$$\begin{aligned} M_{(\mathbf{5},\bar{\mathbf{5}})} &= (Y^\Phi\langle \mathbf{1}_\Phi\rangle, Y^C\langle \mathbf{16}_C\rangle) = \langle \mathbf{1}_\Phi\rangle (Y^\Phi, RY^C), \\ R &\equiv \langle \mathbf{16}_C\rangle / \langle \mathbf{1}_\Phi\rangle \equiv \lambda^r. \end{aligned} \tag{6}$$

As discussed in Ref. [7], the light $\bar{\mathbf{5}}$ fields can be classified by the parameter r. If the VEV $\langle\Phi\rangle$ is not much larger than the VEV $\langle C\rangle$ ($0\le r<3$), then all three main modes of the light $\bar{\mathbf{5}}$ fields come from the first two generation fields, Ψ_1 and Ψ_2. That is, the main light modes become

$\mathbf{16}_{\Psi_1}$, $\mathbf{10}_{\Psi_1}$, and $\mathbf{16}_{\Psi_2}$. Using the basis in which each light mode includes no other main light modes, the light modes can be written as

$$\begin{aligned}\bar{\mathbf{5}}_1 &= \mathbf{16}_{\Psi_1} + \lambda^3 \mathbf{16}_{\Psi_3}[+\lambda^{2+r}\mathbf{10}_{\Psi_2}] + \lambda^{3+r}\mathbf{10}_{\Psi_3}, \\ \bar{\mathbf{5}}_2 &= \mathbf{10}_{\Psi_1} + \lambda^{3-r}\mathbf{16}_{\Psi_3}[+\lambda^2\mathbf{10}_{\Psi_2}] + \lambda^3\mathbf{10}_{\Psi_3}, \\ \bar{\mathbf{5}}_3 &= \mathbf{16}_{\Psi_2} + \lambda^2\mathbf{16}_{\Psi_3} + \lambda^r\mathbf{10}_{\Psi_2} + \lambda^{2+r}\mathbf{10}_{\Psi_3},\end{aligned} \tag{7}$$

where the first terms on the right-hand sides are the main components of these massless modes, and the other terms are mixing terms with the heavy states, $\mathbf{16}_{\Psi_3}$, $\mathbf{10}_{\Psi_2}$, and $\mathbf{10}_{\Psi_3}$. Note that the coefficients of mixings for all three heavy modes are essentially determined by the hierarchical Yukawa couplings Y and R. Further, the $\mathbf{10}_{\Psi_i}$ modes have no Yukawa couplings if the SM doublet Higgs comes from $\mathbf{10}_\Phi$ and $\mathbf{10}_C$. Therefore, the mixings of $\mathbf{16}_{\Psi_3}$ are important to estimate the Yukawa couplings. Then, Yukawa couplings can be calculated as

$$\begin{aligned}Y_u &\sim Y, \\ Y_d &\sim Y_e^T \\ &\sim \begin{pmatrix} Y_{11}+Y_{13}\lambda^3 & Y_{13}\lambda^{3-r} & Y_{12}+Y_{13}\lambda^2 \\ Y_{21}+Y_{23}\lambda^3 & Y_{23}\lambda^{3-r} & Y_{22}+Y_{23}\lambda^2 \\ Y_{31}+Y_{33}\lambda^3 & Y_{33}\lambda^{3-r} & Y_{32}+Y_{33}\lambda^2 \end{pmatrix}.\end{aligned} \tag{8}$$

It is apparent that the Yukawa matrix Y_d has a milder hierarchy than that of Y_u under these conditions. The essential point is that the $\bar{\mathbf{5}}$ fields from Ψ_3, which has larger Yukawa couplings, become superheavy, and the light $\bar{\mathbf{5}}$ fields come from the first two generation fields Ψ_1 and Ψ_2. This results in a small $\tan\beta \equiv \langle H_u\rangle / \langle H_d\rangle$, which is roughly estimated as $\tan\beta \sim (m_t/m_b)(Y_{32}/Y_{33})$ up to renormalization group effects, because it is expected that $(Y_{32}/Y_{33})^2 \leq m_c/m_t$.

If we take $r = 0.5$, the Yukawa matrices become

$$Y_d \sim Y_e^T \sim \begin{pmatrix} 0[\lambda^6] & 0[\lambda^{5.5}] & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}, \tag{9}$$

which give almost realistic masses of down quarks and charged leptons when $\tan\beta \sim (m_t/m_b)\lambda^2$ up to renormalization effects. The CKM matrix can be calculated as

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{10}$$

In order to avoid the unrealistic $SU(5)$ GUT relation, $Y_d = Y_e^T$, we have to pick up the VEVs of A in these Yukawa matrices. In principle, we can pick up the effects in the mass matrix of $\mathbf{5}$ and $\bar{\mathbf{5}}$ and/or Yukawa interactions by the higher dimensional interactions. Here, we simply assume it in order to obtain realistic quark and lepton mass matrices. This point is discussed in the following sections.

NEUTRINO MASSES AND MIXINGS

Because representation $\mathbf{27}$ has two singlets N_R^c and S under the SM gauge group, the Dirac mass matrix becomes 3×6. The Dirac neutrino mass matrix is obtained from the interactions (3) as

$$Y_{\nu_D} = (Y_N, Y_S) \sim (1, \lambda^r) \otimes \begin{pmatrix} 0[\lambda^6] & \lambda^5 & \lambda^3 \\ 0[\lambda^{6-r}] & \lambda^{5-r} & \lambda^{3-r} \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix} \tag{11}$$

Because the Dirac neutrino masses become λ^2 smaller than the usual $SO(10)$ GUT predictions $Y_u \sim Y_{\nu_D}$, the smaller right-handed neutrino masses are required to realize the correct neutrino mass scales corresponding to the observed neutrino oscillations. (Note that this difference is important in suppressing FCNC processes (e.g. $\mu \to e\gamma$) that originate from loop corrections to SUSY breaking parameters [11], because such corrections are proportional to $Y_{\nu_D}^\dagger Y_{\nu_D}$.) The right-handed neutrino mass matrix (6×6) is obtained from the interactions

$$Y_{ij}^{\bar{X}\bar{Y}} \Psi_i \Psi_j \frac{\bar{X}\bar{Y}}{\Lambda}, \tag{12}$$

where $\bar{X}, \bar{Y} = \bar{\Phi}, \bar{C}$, as

$$\begin{aligned}M_{\nu_R} &= \begin{pmatrix} Y^{\bar{\Phi}\bar{\Phi}}\langle\bar{\Phi}\rangle^2 & Y^{\bar{\Phi}\bar{C}}\langle\bar{\Phi}\rangle\langle\bar{C}\rangle \\ Y^{\bar{\Phi}\bar{C}}\langle\bar{\Phi}\rangle\langle\bar{C}\rangle & Y^{\bar{C}\bar{C}}\langle\bar{C}\rangle^2 \end{pmatrix}\frac{1}{\Lambda}, \\ &\sim \begin{pmatrix} 1 & \lambda^r \\ \lambda^r & \lambda^{2r}\end{pmatrix} \otimes \begin{pmatrix} 0[\lambda^6] & \lambda^5 & 0[\lambda^3] \\ \lambda^5 & \lambda^4 & \lambda^2 \\ 0[\lambda^3] & \lambda^2 & 1 \end{pmatrix} \frac{c\langle\bar{\Phi}\rangle^2}{\Lambda}\end{aligned} \tag{13}$$

Here, we take $Y^{\bar{X}\bar{Y}} \sim cY^\Phi$, where c is a constant. Note that the smallness of the right-handed neutrino masses is naturally expected in this scenario, because the right-handed neutrino masses are obtained from the higher dimensional interactions. Then, the light neutrino mass matrix is obtained by seesaw mechanism [21] as

$$\begin{aligned}M_\nu &= Y_{\nu_D} M_{\nu_R}^{-1} Y_{\nu_D}^T \langle H_u\rangle^2 \eta^2 \\ &\sim \lambda^4 \begin{pmatrix} \lambda^2 & \lambda^{2-r} & \lambda \\ \lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{pmatrix} \frac{\Lambda\langle H_u\rangle^2\eta^2}{c\langle\bar{\Phi}\rangle^2},\end{aligned} \tag{14}$$

where η is a renormalization parameter. If we take $\langle\Phi\rangle \sim 10^{16}$ GeV, $\Lambda \sim 10^{19}$ GeV, $\langle H_u\rangle\eta \sim 100$ GeV, $c \sim 0.1$, and $r \sim 0.5$, we can obtain realistic neutrino masses and the Maki-Nakagawa-Sakata (MNS) matrix [22] as

$$V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \tag{15}$$

NON-ABELIAN HORIZONTAL SYMMETRY

One of the most attractive features in this E_6 GUT is that non-Abelian horizontal symmetry can be natu-

rally introduced to solve the SUSY flavor problem. One of the important points in the above scenario is that all the realistic mass hierarchies of quarks and leptons can be obtained from just one basic hierarchical structure of the Yukawa matrix, Y, which can be naturally obtained from one non-Abelian horizontal symmetry. (Usually, to obtain various hierarchies of quarks and leptons, several non-Abelian horizontal symmetries are introduced or the coefficients are tuned to realize various hierarchies from one horizontal symmetry, or more Higgs fields whose VEVs break the horizontal symmetry are introduced.)

Usually, $SU(2)_H$ or $U(2)_H$ horizontal symmetry is adopted because it realizes the universal sfermion masses of the first two generation fields that are doublets under the horizontal symmetry, which is important in solving the SUSY flavor problem, and because top Yukawa coupling is allowed by the symmetry when the third generation fields and the Higgs fields are singlets under the horizontal symmetry.

However, the universal sfermion masses of the first two generation fields, $\tilde{m}^2_{\bar{\mathbf{5}}} \sim \mathrm{diag}(\tilde{m}^2, \tilde{m}^2, (a+1)\tilde{m}^2)$, are not enough to suppress the FCNC processes in the GUT models in which diagonalizing matrices V_x $x = \bar{\mathbf{5}}, \mathbf{10}$ can be estimated as $V_{\mathbf{10}} \sim V_{CKM}$ and $V_{\bar{\mathbf{5}}} \sim V_{MNS}$ as in the previous models. This is because the mixing matrices defined as $\delta_x \equiv V_x^\dagger \frac{\tilde{m}_x^2 - \tilde{m}^2}{\tilde{m}^2} V_x$[23] become

$$\delta_{\bar{\mathbf{5}}} \sim V_{\bar{\mathbf{5}}}^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix} V_{\bar{5}} \sim \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} a \quad (16)$$

at the GUT scale, which does not satisfy the constraints of various FCNC processes;

$$\begin{aligned} \sqrt{|\mathrm{Im}(\delta_{D_L})_{12}(\delta_{D_R})_{12})|} &\leq 2\times 10^{-4}\left(\frac{\tilde{m}_Q}{500\ \mathrm{GeV}}\right) \\ |\mathrm{Im}(\delta_{D_R})_{12}| &\leq 1.5\times 10^{-3}\left(\frac{\tilde{m}_Q}{500\ \mathrm{GeV}}\right) \end{aligned} \quad (17)$$

at the weak scale from ϵ_K in K meson mixing, and

$$|(\delta_{E_L})_{12}| \leq 4\times 10^{-3}\left(\frac{\tilde{m}_L}{100\ \mathrm{GeV}}\right)^2 \quad (18)$$

from the $\mu \to e\gamma$ process.

It is interesting that in the E_6 unification, such a problem is naturally solved. As discussed in the previous section, in E_6 GUT, it is natural that all the three light $\bar{\mathbf{5}}$ fields come from the first two generation fields. Therefore, if we introduce horizontal symmetry $SU(2)_H$ or $U(2)_H$, all the sfermion masses of $\bar{\mathbf{5}}$ fields become equivalent in the leading order, that is, $a = 0$. Note that the structure also realizes bi-large neutrino mixings as discussed in the previous section. It is suggestive that the same structure solves the FCNC problem that originate from the large neutrino mixings.

Of course, the horizontal symmetry has to be broken to obtain the realistic quark and lepton mass matrices, and this effect also breaks the universality of the sfermion masses. Next, we examine this breaking effect in a concrete model.

We introduce a global horizontal symmetry $U(2)_H$ and fields listed in Table I.

Table I. Odd R-parity for matter fields Ψ and Ψ_3 is introduced.

	Ψ	Ψ_3	$\bar{F}$	Θ	A	Φ	$\bar{\Phi}$	C	$\bar{C}$
E_6	**27**	**27**	**1**	**1**	**78**	**27**	$\overline{\mathbf{27}}$	**27**	$\overline{\mathbf{27}}$
$SU(2)_H$	**2**	**1**	$\bar{\mathbf{2}}$	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_H$	1	0	-1	-2	0	0	0	0	0

Here, we simply assume that $U(2)_H$ is broken by the VEVs of $\bar{F}$ and Θ as

$$U(2)_H \underset{\langle\Theta\rangle\sim\lambda}{\longrightarrow} SU(2)_H \underset{\langle\bar{F}_a\rangle\sim\lambda^2\delta_a^2}{\longrightarrow} \text{nothing}, \quad (19)$$

and E_6 is broken into the SM gauge group by the VEVs of Φ, $\bar{\Phi}$, C, $\bar{C}$, and A;

$$\begin{aligned} \langle \mathbf{1}_\Phi \rangle &= \langle \mathbf{1}_{\bar{\Phi}} \rangle \sim \lambda^{3.5}, & (20) \\ \langle \mathbf{16}_C \rangle &= \langle \overline{\mathbf{16}}_{\bar{C}} \rangle \sim \lambda^4, & (21) \\ \langle A \rangle &\sim \lambda^4. & (22) \end{aligned}$$

In this paper, we sometimes use a unit in which $\Lambda = 1$. Then, the type A basic hierarchical Yukawa matrix Y is obtained from the interactions

$$((\Psi_3)^2 + \Psi_3\Psi\bar{F} + (\Psi\bar{F})^2 + \Psi A\Psi\Theta)(\Phi + C). \quad (23)$$

Note that the terms $\Psi\Psi\Theta(\Phi + C)$ become trivially zero. The light $\bar{\mathbf{5}}$ fields are obtained as in Eq. (7) with $r = 0.5$. Here, for simplicity, H_d comes from a linear combination $\mathbf{10}_\Phi + \mathbf{10}_C$. (If both SM Higgs H_d and H_u come from only $\mathbf{10}_\Phi$, CKM mixing becomes too small because of a cancellation.) Then, we obtain a realistic quark and lepton mass matrices, including bi-large neutrino mixings, as discussed in the previous section. Here, the interaction $\Psi A\Psi\Theta(\Phi + C)$ plays an important role in avoiding unrealistic $SU(5)$ GUT relations $Y_d = Y_e^T$. Generically, the VEV of A gives different contributions to the Yukawa couplings of quarks and leptons; therefore, the quark mass matrices can be different from the lepton mass matrices. Note that the mixing coefficients of D_R^c in Eq. (7) are of the same order as those of L but have generically different values from those of L.

Note that the main modes of the light $\bar{\mathbf{5}}$ fields are obtained from the first two generation fields, which come from a single field Ψ in this case. Therefore, unless the horizontal symmetry is broken, the sfermion masses are obtained as

$$\tilde{m}^2_{\mathbf{10}} \sim \tilde{m}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & O(1) \end{pmatrix}, \quad (24)$$

$$\tilde{m}^2_{\bar{\mathbf{5}}} \sim \tilde{m}^2 \begin{pmatrix} 1+\lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^{5.5} & 1+\lambda^5 & \lambda^{4.5} \\ \lambda^5 & \lambda^{4.5} & 1+\lambda^4 \end{pmatrix}, \quad (25)$$

where the corrections to the sfermion masses $\tilde{m}_{\bar{\mathbf{5}}}$ come from the mixings in Eq. (7). When the breaking of the horizontal symmetry is taken into account, the sfermion mass matrices are corrected as

$$\Delta\tilde{m}^2_{\mathbf{10}} \sim \tilde{m}^2 \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & O(1) \end{pmatrix}, \quad (26)$$

$$\Delta\tilde{m}^2_{\bar{\mathbf{5}}} \sim \tilde{m}^2 \begin{pmatrix} \lambda^4 & \lambda^{6.5} & \lambda^5 \\ \lambda^{6.5} & \lambda^4 & \lambda^{4.5} \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \end{pmatrix}, \quad (27)$$

which are calculated mainly from the interactions

$$\int d^2\theta d^2\bar{\theta} \quad (\quad (\Psi_3 + \Psi\bar{F} + \Psi\bar{F}^\dagger\Theta)^\dagger(\Psi_3 + \Psi\bar{F} + \Psi\bar{F}^\dagger\Theta) + \Psi^\dagger A\Psi + \Psi_3^\dagger A\Psi_3)\frac{X^\dagger X}{\Lambda^2} \quad (28)$$

where X is a spurion field whose VEV of F-term is given as $F_X = \tilde{m}\Lambda$. The last two terms in Eq. (28) splits the masses of scalar down quarks included in fields **10** and **16** because they have different $B-L$ charges. Such effects is important in estimating the corrections to sfermion masses because only the main mode of $\bar{\mathbf{5}}_2$ comes from **10** of $SO(10)$ and the other main modes come from **16**.

The mixing matrices $\delta_x \equiv V_x^\dagger \Delta\tilde{m}_x^2 V_x/\tilde{m}^2$ ($x = \bar{\mathbf{5}}, \mathbf{10}$) are approximated as

$$\delta_{\mathbf{10}} = \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & O(1) \end{pmatrix}, \; \delta_{\bar{\mathbf{5}}} = \begin{pmatrix} \lambda^4 & \lambda^{4.5} & \lambda^5 \\ \lambda^{4.5} & \lambda^4 & \lambda^{4.5} \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \end{pmatrix} \quad (29)$$

at the GUT scale. The constraints (17) at the weak scale from ϵ_K in K meson mixing require scalar quark masses larger than 300 GeV, because in this model $\sqrt{|(\delta_{D_L})_{12}(\delta_{D_R})_{12})|} \sim \lambda^{4.75}(\eta_q)^{-1}$ and $|(\delta_{D_R})_{12}| \sim \lambda^{4.5}(\eta_q)^{-1}$, where we take a renormalization factor $\eta_q \sim 6$ [30]. Further, the constraint from the $\mu \to e\gamma$ process in Eq. (18) is easily satisfied, because $|(\delta_{E_L})_{12}| \sim \lambda^{4.5}$ in this model.

It is quite impressive that the structure which realizes bi-large neutrino mixings simultaneously solves the SUSY FCNC problem in non-Abelian horizontal symmetry that originates from large neutrino mixings.

If the spurion field X has interactions in the superpotential

$$((\Psi_3)^2 + \Psi_3\Psi\bar{F} + (\Psi\bar{F})^2 + \Psi A\Psi\Theta)(\Phi + C)X, \quad (30)$$

the left-right mixings in sfermion masses Δ_X^{LR} ($X = U, D, L$) are induced. In the above models, $\delta_X^{LR} \equiv \Delta_X^{LR}/\tilde{m}_X^2$ are calculated as

$$\delta_D^{LR} \sim \delta_L^{RL} \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix} \frac{\langle H_d \rangle}{\tilde{m}_X}, \quad (31)$$

$$\delta_U^{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \frac{\langle H_u \rangle}{\tilde{m}_U}. \quad (32)$$

The constraints for these mixings

$$|\mathrm{Im}(\delta_D^{LR})_{12}| < 2\times 10^{-5} \times \left(\frac{\tilde{m}_d}{500\ \mathrm{GeV}}\right)^2, \quad (33)$$

$$|(\delta_D^{LR})_{23}| < 1.6\times 10^{-2} \times \left(\frac{\tilde{m}_d}{500\ \mathrm{GeV}}\right)^2, \quad (34)$$

$$|(\delta_L^{LR})_{12}| < 1\times 10^{-6} \times \left(\frac{\tilde{m}_l}{100\ \mathrm{GeV}}\right)^2, \quad (35)$$

$$|(\delta_L^{LR})_{23}| < 6\times 10^{-3} \times \left(\frac{\tilde{m}_l}{100\ \mathrm{GeV}}\right)^2, \quad (36)$$

which are obtained from ϵ'/ϵ, $b \to s\gamma$, $\mu \to e\gamma$, and $\tau \to \mu\gamma$ processes, respectively [23], require roughly $\tilde{m}_d > 1$ TeV and $\tilde{m}_l > 500$ GeV. These constraints are not so severe especially in our scenario, because the sfermion masses can be large except those of the third generation $\mathbf{10}_3$ of $SU(5)$ that must be small to stabilize the weak scale. Moreover, a reasonable assumption like SUSY breaking in the hidden sector, can lead to vanishing δ^{LR} [27]. Actually, if the SUSY breaking sector is separated from the visible sector in the superpotential, the interactions (30) are forbidden.

Because there is a strong suspicion that global symmetries are broken through quantum gravitational effects, it may be more important to use local symmetries instead of the global symmetries. For example, we can regard the $SU(2)_H \times U(1)_H$ symmetry in Table I as local symmetry, if we add a field $F(\mathbf{2}, \mathbf{1})$ whose VEV is given as $|\langle F \rangle| = |\langle \bar{F} \rangle|$ to satisfy the D-flatness condition of $SU(2)_H$. (Here, the D-flatness condition of $U(1)_H$ seems not to be satisfied; however, the Fayet-Illiopoulos D-term, if any, can improve the situation. Further, $U(1)_H$ may be anomalous $U(1)$, whose anomaly is cancelled by Green-Schwarz mechanism [24].) This modification changes the basic structure of Yukawa couplings into type B, but the mixing matrices δ_x has no essential difference. However, generally, the D-term of $SU(2)_H$ has non-vanishing VEV, which may be another source to break the universality of the sfermion masses. Therefore, we must assume that the D of $SU(2)_H$ is sufficiently small due to some mechanism. For example, in the superstring theory, if modular weights of F and $\bar{F}$ are equivalent, the SUSY breaking masses for the scalar components of F and $\bar{F}$ become equal, which realizes the vanishing D of $SU(2)_H$ [25].

It is straightforward to extend the horizontal symmetry $SU(2)_H$ to $SU(3)_H$ in which all the three family quark and leptons can be unified into a single multiplet $(\mathbf{27}, \mathbf{3})$ under $E_6 \times SU(3)_H$ as discussed in Ref. [10]. Because $SU(3)_H$ must be broken around the cutoff scale to obtain large top Yukawa coupling, mass matrices of sfermion have no essential difference from those of $SU(2)_H$ models.

In either of the horizontal symmetries, this scenario predicts a special pattern of sfermion masses. That is, all the sfermion masses are universal except those of the third generation $\mathbf{10}_3 = (Q, U_R^c, E_R^c)$ fields around the GUT scale. This prediction can be tested by a linear collider in the future.

DISCUSSION

In this paper, we did not discuss methods to determine the VEVs of the Higgs which break $E_6 \times U(2)_H$ into the SM gauge group. Therefore, we have simply determined the scales of non-vanishing VEVs of fields F, $\bar{F}$, A, Φ, $\bar{\Phi}$, C, and $\bar{C}$, and the SM Higgs mixings in order that realistic (scalar) fermion masses and mixings are obtained. The SM Higgs mixing is important to avoid a cancellation of CKM mixings. The VEV of $\bar{F}$ is determined in order to satisfy $\langle \bar{F} \rangle \sim \sqrt{m_c/m_t} \sim \lambda^2$ at the GUT scale. It is interesting that this value is enough to satisfy the various FCNC constraints. The ratio $R \equiv \langle C \rangle / \langle \Phi \rangle \sim \lambda^{0.5}$ is important to obtain bi-large neutrino mixings, although $R \sim \lambda$ also gives a realistic pattern of quark and lepton mass matrices. Because the E_6 structure is important in solving the SUSY flavor problem, the VEVs $\langle \Phi \rangle$, $\langle C \rangle$, or A, which break E_6, cannot be taken as very large values in order to suppress FCNC processes.

If $\langle \mathbf{45}_A \rangle \sim \lambda^4 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \mathrm{diag}(1,1,1,0,0) \propto Q_{B-L}$, which sometimes plays an important role in solving doublet-triplet splitting problem, is adopted, the mixings of lepton doublet L in Eq. (7) are drastically changed in the type A model used in this paper, because the L and $\bar{L}$ fields in $\mathbf{10}$ of $SO(10)$ have vanishing Q_{B-L} charges. In this case, other contributions to the basic Yukawa coupling Y are required. (For example, the type B model, in which the horizontal symmetry is localized, does not change the basic arguments in this paper because of other contributions.)

It is obvious that our arguments can be applied to any model in which the appropriate scales of VEVs are obtained; in other words, the appropriate coefficients in the Higgs potential are obtained. Therefore, it is interesting to examine which among the various mechanisms that can determine coefficients of interactions are suitable for our arguments. Such a project is important but is beyond the scope of this paper.

This is a promising way [5, 7, 12, 26] to introduce an anomalous $U(1)_A$ symmetry (or just $U(1)$ with Fayet-Illiopoulos D-term) to control the Higgs sector. This is because the scales of the various non-vanishing VEVs and the SM Higgs mixings are determined by their $U(1)_A$ charges, for example, $\langle A \rangle \sim \lambda^{-a}$ (Here, we use small letter for their charges). Such determination of VEVs plays an important role in defining the effective charges that determine all the orders of coefficients. Further, because the interactions including A (for example, $\lambda^{\psi_i+\psi_j+a+\phi}\Psi_i A \Psi_j \Phi$) automatically gives the same contributions to the Yukawa couplings as the interaction without A ($\lambda^{\psi_i+\psi_j+\phi}\Psi_i \Psi_j \Phi$), the unrealistic GUT relations between fermion mass matrices can be naturally avoided. Moreover, such VEV relations guarantee the natural gauge coupling unification [26], though it requires a rather small cutoff scale such as $\Lambda \sim 2 \times 10^{16}$ GeV. Actually, using anomalous $U(1)_A$ symmetry, we can obtain a complete GUT with $E_6 \times SU(3)_H$ or $E_6 \times SU(2)_H$ gauge symmetry [10, 12], in which doublet-triplet splitting, realistic quark and lepton mass matrices, natural gauge coupling unification, and suppressed FCNC are realized. Although the suppression of FCNC becomes milder than the concrete model discussed in this paper because of the lower cutoff, it is interesting to note that such models can be built with generic interactions (including higher dimensional interactions) with $O(1)$ coefficients.

E_6 symmetry is sufficient, but not necessary to realize the interesting structure discussed in this paper. $SU(2)_E$ [6] is the essential symmetry, which rotates two $\bar{\mathbf{5}}$s and two $\mathbf{1}$s of $SU(5)$ in a $\mathbf{27}$ of E_6 as a doublet. Therefore, if a gauge group that includes $SU(2)_E$, (for example, $SU(3)^3$ [28], $SU(6) \times SU(2)_E$, and flipped $SO(10)$ $[SO(10)' \times U(1)]$ [29]) is adopted, then the arguments in this paper can be applied.

SUMMARY

In this paper, we pointed out that in E_6 GUT, once a basic hierarchical structure of Yukawa couplings is given, larger neutrino mixings than the CKM mixings are naturally realized. This is independent of the origin of the hierarchy. Therefore, this mechanism can be applied to various models in which the hierarchical structure of Yukawa couplings is realized.

Moreover, we pointed out that non-Abelian horizontal symmetry is a promising candidate for the origin of the hierarchy, which can solve the SUSY flavor problem. This is because only one hierarchical structure, which is easily obtained by introducing a non-Abelian horizontal symmetry, induces all the other hierarchical structures of quark and lepton mass matrices. It is non-trivial that in E_6 GUT, the structure, which realizes bi-large neutrino mixings, also realizes universal sfermion masses for all three generation $\bar{\mathbf{5}}$ fields that are important in suppressing FCNC processes naturally in GUT theory with bi-large neutrino mixings.

It is quite impressive that unification of quark and leptons realizes larger neutrino mixings and solves the SUSY flavor problem in E_6 unification. We hope that this compatibility is an evidence of E_6 GUT or the E_6 structure in our world and the characteristic pattern of sfermion masses is confirmed in future experiments.

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